MATH 170 – CHAPTER 4 Name: .

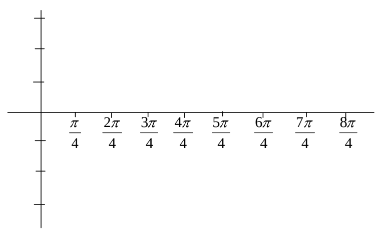
4.1 Graphs of Trig Functions

Need To Know

- Graph one cycle of the 3 trig functions
- How to sketch a graph of a trig function
- Definitions for Even and Odd Functions
- Study Hard in Chapter 4

y = sin x

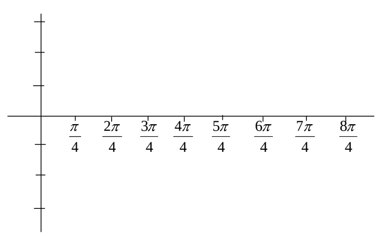
x	y
0	0
π/6	0.5
π/4	0.7
π/3	0.86
2π/4	1
3π/4	0.7
4π/4	0
5π/4	-0.7
6π/4	-1
7π/4	-0.7
8π/4	0



Facts and Observations

y = cos x

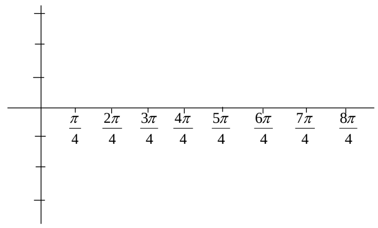
x	y
0	1
π/6	0.86
π/4	0.7
π/3	0.5
2π/4	0
3π/4	-0.7
4π/4	-1
5π/4	-0.7
6π/4	0
7π/4	0.7
8π/4	1



Facts and Observations

y = tan x

x	y
0	0
π/4	1
π/3	1.7
2π/4	***
2π/3	-1.7
3π/4	-1
4π/4	0
5π/4	1
4π/3	1.7
6π/4	***
5π/3	-1.7
7π/4	-1
8π/4	0



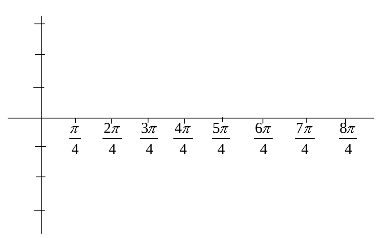
Facts and Observations

Review Windows for Sketches

Sine Window Cosine Window Tangent Window

y = cot x

x	tan x	cot x
0	0	***
π/4	1	1
π/3	1.7	.6
2π/4	***	0
2π/3	-1.7	-.6
3π/4	-1	-1
4π/4	0	***
5π/4	1	1
4π/3	1.7	.6
6π/4	***	0
5π/3	-1.7	-.6
7π/4	-1	-1
8π/4	0	***



Facts and Observations

Down only

Cycle = π on (0, π)

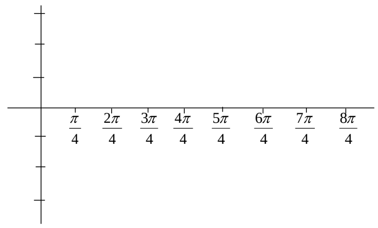
Max & min is N/A

Domain is all reals except x ≠ kπ

Range is all real numbers

y = sec x

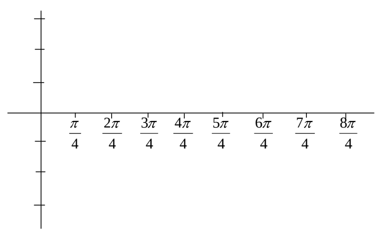
x	cos x	sec x
0	1	1
π/4	0.7	1.4
π/3	0.5	2.0
2π/4	0	****
2π/3	-0.5	-2.0
3π/4	-0.7	-1.4
4π/4	-1	-1
5π/4	-0.7	-1.4
4π/3	-0.5	-2.0
6π/4	0	****
5π/3	0.5	2.0
7π/4	0.7	1.4
8π/4	1	1



Facts and Observations

y = csc x

x	sin x	csc x
0	0	****
π/6	0.5	2.0
π/4	0.7	1.4
2π/4	1	1
3π/4	0.7	1.4
5π/6	0.5	2.0
4π/4	0	****
7π/6	-0.5	-2.0
5π/4	-0.7	-1.4
6π/4	-1	-1
7π/4	-0.7	-1.4
11π/6	-0.5	-2.0
8π/4	0	****



Facts and Observations

Sketch sine

Asymptotes where sine is 0; at x = 0, π, 2π

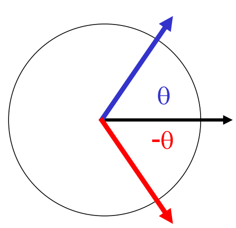
Sketch reciprocal curve above & below sine wave

Even and Odd Functions

Definitions:

An *even function* is a function for which f(-x) = f(x) for all x in the domain of f.

An *odd function* is a function for which f(-x) = -f(x) for all x in the domain of f.



Simplify:

cos(θ) tan(-θ)

New Identities:

____

____

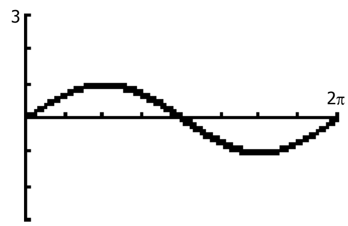
4.2 Parameters of Trig Functions

<u>Need To Know</u>

- $A\sin(Bx)$ or $A\cos(Bx)$
  1. Understand the effect of A
  2. Understand the effect of B
- Define and relate to graphs: Reflection, Amplitude, and Period
- Calculate Amplitude and Period
- Quick Sketch of:
  1. One cycle of Sine or Cosine
  2. Multiple cycles of Sine or Cosine

Observe $y = A\sin(Bx)$

$y_1 = \sin(x)$



$y_2 = 3\sin(x)$

$y_3 = (1/2)\sin(x)$

The "A" corresponds to ____________________.
It's effect ______________________ the vertical.
A = __________________

Graphing $y = A\sin(Bx)$

$y_1 = \sin(x)$

$y_2 = -\sin(x)$

$y_3 = -3\sin(x)$

Make a prediction before seeing graph.

The effect of negative "A" ______________

Graphing "B"   y = A sin(Bx)

$y_1 = \sin(x)$

$y_2 = \sin(2x)$

Describe the effect of "B"

B = 1 → 1 cycle on 2π   cycle length = 2π

B = __→ __ cycles on 2π   cycle length = __

Graphing y = A sin(Bx)

$y_1 = \sin(x)$

$y_2 = \sin(3x)$

What is a formula for the cycle length?

B = __

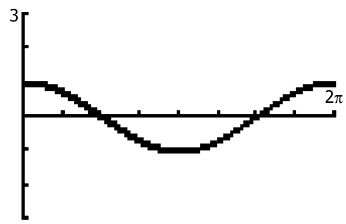
__ cycles on 2π

cycle length = __

Graphing y = A cos(Bx)

$y_1 = \cos(x)$

$y_2 = \cos(4x)$



Does the formula work for cosine?

B = __

__ cycles on 2π

cycle length = __

Summarize

|A| = ________________

(Negative A causes a reflection or flip)

B = ________________

_______ = cycle length on one cycle

Period = __ for sine, cosine, secant, cosecant.

Period = __ for tangent or cotangent.

4.3 Phase Shift

<u>Need To Know</u>

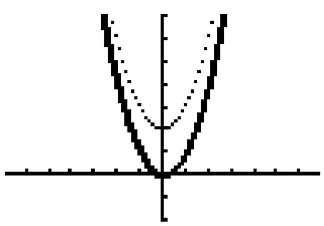
- A sin(Bx + C) + D or A cos(Bx+ C) + D
  - Understand the effect of C
  - Understand the effect of D
- Define and relate to graphs: Translation (Vertical Shift) and Phase Shift
- Calculate Vertical Shift and Phase Shift
- Quick Sketch of:
  - One cycle of Sine or Cosine
  - Multiple cycles of Sine or Cosine

Translation – The Effect of "D"

Graph $f(x) = x^2$

What's the effect of adding a constant?

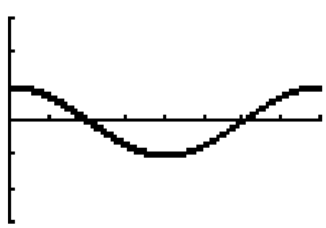
Example: $g(x) = x^2 + 2$



Predictions:

$y_1 = \cos x$

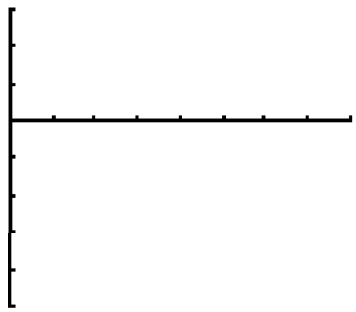
$y_2 = \cos x + 2$?



Quick Sketch – The Effect of "D"

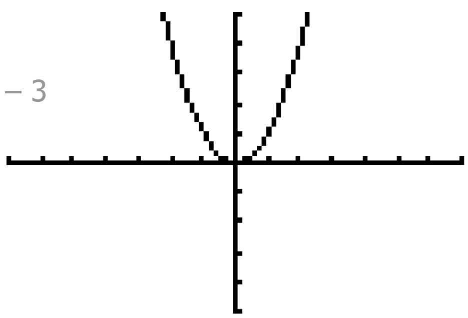
Sketch the graph without a calculator:
y = 3cos 2x – 1 on [0, 2π]

Trans =
Amp =
Cycles=
Per =
Draw window



Graphing Functions

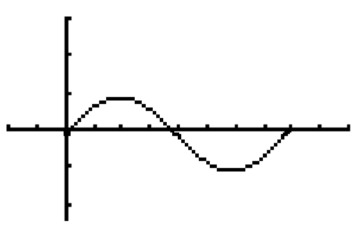
Recall: $f(x) = x^2$
What is the effect of adding a constant?
$g(x) = x^2 - 3$
$h(x) = (x - 1)^2$
$k(x) = (x - 1)^2 - 3$



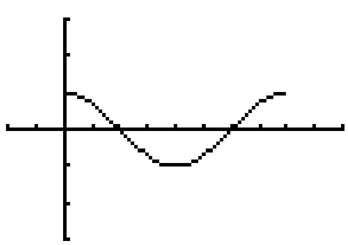
Explore the Effect of "C"

Graph :

$$y = \sin\left(x + \frac{\pi}{4}\right)$$



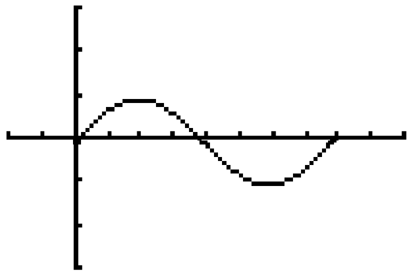
$$y = 2\cos\left(x - \frac{\pi}{2}\right)$$



Explore the Effect of "C"

Graph :

$y=-3\sin\left(2x-\frac{\pi}{2}\right)$



Summarize

Amplitude = ____ (Negative A causes a reflection or flip)

Period = __ or Period = $\frac{\pi}{B}$ for tangent or cotangent.

Phase Shift = ___

Vertical Shift =

y = sin(argument) or cos(argument)

The ______________________________

Practice

Find the amplitude, period and phase shift. Do NOT graph.

$y=-\cos\left(2x-\frac{\pi}{3}\right)$

Practice

How Graph WITHOUT a Calculator

1) Get data from the equation
2) Use **the argument** to find the starting and ending of cycle
3) Draw the width of box (period)
4) Draw the height of the box (amp)
5) Label x axis with tixs for quarter points
6) Label y axis
7) Draw in target points and Sketch wave
8) Add additional cycles if needed

Sketch one cycle of

$$y = -1 + \sin(3x + \pi)$$

Practice

How Graph WITHOUT a Calculator

1) Get data from the equation
2) Use **the argument** to find the starting and ending of cycle
3) Draw the width of box (period)
4) Draw the height of the box (amp)
5) Label x axis with tixs for quarter points
6) Label y axis
7) Draw in target points and Sketch wave
8) Add additional cycles if needed

Sketch on $\left[\frac{-5\pi}{2}, \frac{7\pi}{2}\right]$

Clearly label so amplitude, period, phase shift and x-axis increments are indicated.

$$y = -4\cos(2x + \frac{\pi}{2})$$

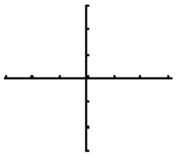
4.4 The Other Trig Function

Need To Know

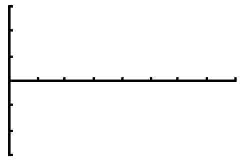
- Graph Other Trig functions w/out calculator
  1. y = Atan(Bx + C) + D
  2. y = Acot(Bx + C) + D
  3. y = Asec(Bx + C) + D
  4. y = Acsc(Bx + C) + D

Review Windows for Sketches

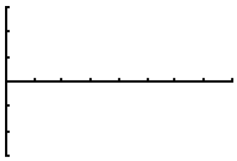
Tangent Window



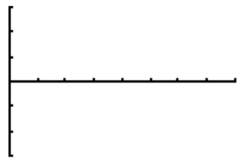
Cotangent Window



Secant Window



Cosecant Window

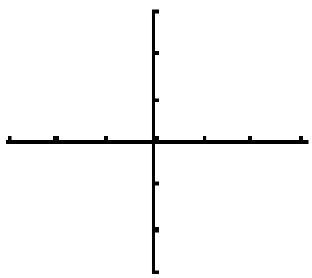


The Effect of "A" on Tan & Cot

Compare

y_1 = tan(x)

y_2 = 4tan(x)

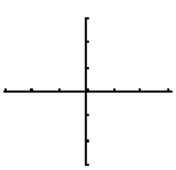


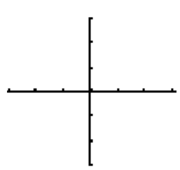
Quick Sketch – No Calculator

Sketch one cycle without a calculator:

y = 3tan(2x)

$$y = \frac{1}{2}\cot\left(\frac{\pi}{2}x\right)$$

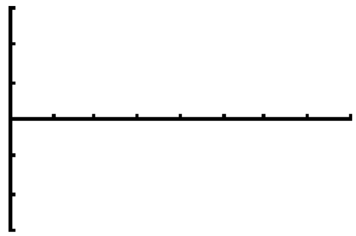




Quick Sketch – No Calculator

Sketch one cycle without a calculator:

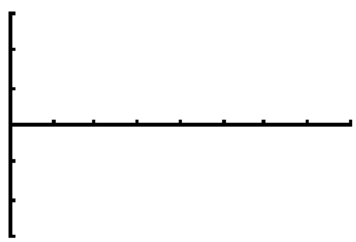
y = -2csc(x)



Quick Sketch – No Calculator

Sketch one cycle without a calculator:

y = $-1+2\sec\left(\pi x+\frac{\pi}{4}\right)$



4.5 Finding the Equation

<u>Need To Know</u>

- Review the basic form of a trig equation
- Finding data from a graph
- (Note: Skip 4.6)

Recall Basic Trig Equation

y = A sin (Bx + C) + D

Amplitude =

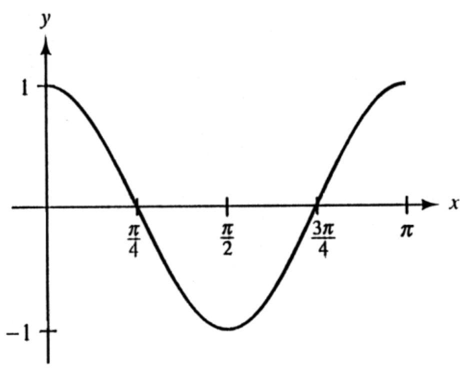
Period =

Phase Shift =

Vertical Shift =

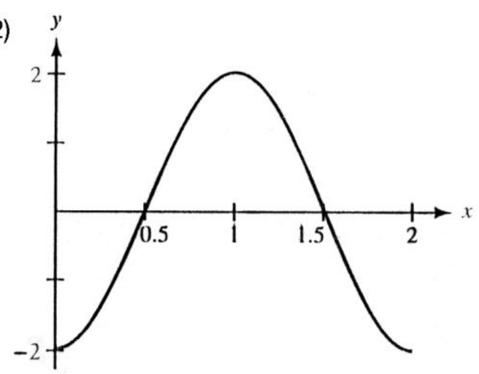
Find the Equations of Graph

1)



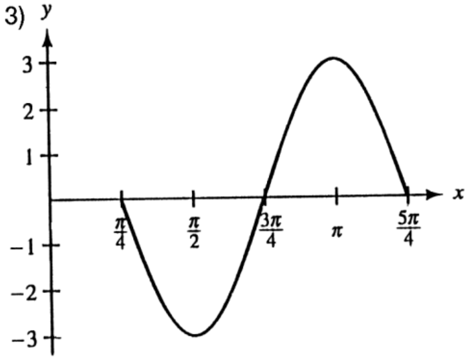
Find the Equations of Graph

2)



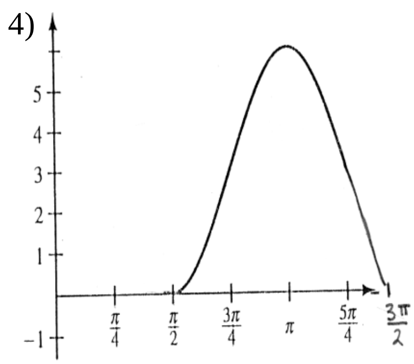
Find the Equations of Graph

3)



Find the Equations of Graph

4)



4.7 Inverse Trig Functions

<u>Need To Know</u>

- Idea of functions and inverse function
- Graphs of inverse trig functions
- Evaluate expressions with inverse trig functions
- Long lesson – write fast and **<u>think</u>** fast

Review Functions Stuff

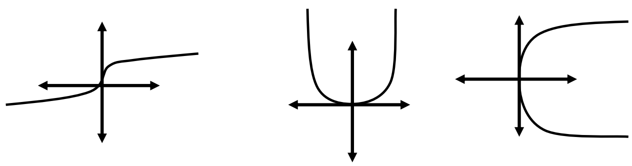
Function – is a rule that assigns one y output for every x input.

________ – is the set of all _____ for the functions.

_______ – is the set of all ______ for the functions.

How do you test a graph to determine if the graph can be written as a function?

____



Inverse Functions Stuff

Inverse of Function – is a rule that reverses or interchanges the pairs. It may not be a function.
Consider ordered pairs in a functions:
What happens to (2,7) & (a,w) in the inverse?

An **Inverse is also a function** only when the original function is a **one-to-one function.**

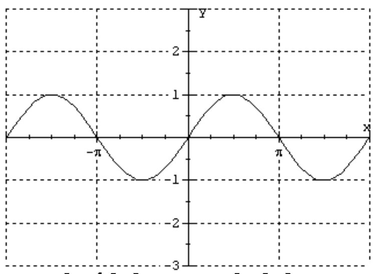
A function must pass the ________________ to have an inverse function.

Inverse Functions Stuff

Observations on Inverse Function

1. If f(x) is function, then the notation $f^{-1}(x)$ is the inverse function (if it exists.)
2. If (x, y) is a pair for f, then ________ is a pair for f^{-1}
3. If D_f is the domain of f and R_f is the range of f, then the domain of f^{-1} is _________________ and the range of f^{-1} is __________________.
4. Given the graph of f(x), then the graph of $f^{-1}(x)$ is ____
5. To find $f^{-1}(x)$ algebraically you must ____________ ____

Inverse of Sine



$y = \sin^{-1}(x) = \arcsin(x)$

Domain of sine inverse is _________.

Range of sine inverse is __________.

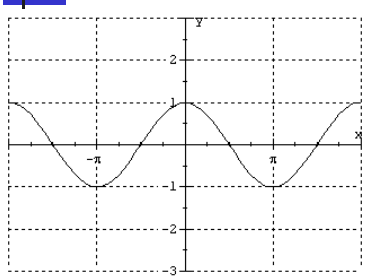
θ is restricted to __________.

Does y = sin x have an inverse functions?

What domain restriction allows for an inverse function?

Restrict sine to _________ then the inverse of sine exists.

Inverse of Cosine



$y = \cos^{-1}(x) = \arccos(x)$

Domain of cosine inverse is _________.

Range of cosine inverse is __________.

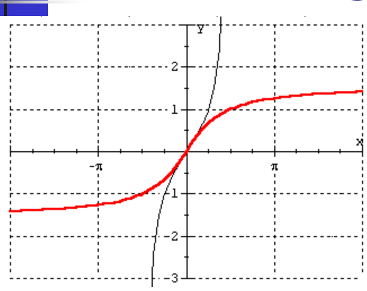
θ is restricted to __________.

Does y = cos x have an inverse functions?

What domain restriction allows for an inverse function?

Restrict cosine to _______ then the inverse of cosine exists.

Inverse of Tangent



$y = \tan^{-1}(x) = \arctan(x)$

Domain of tangent inverse is ________

Range of tangent inverse is ________

θ is restricted to _________

Does y = tan x have an inverse functions?

What domain restriction allows for an inverse function?

Restrict tan to __________ then the inverse of tan exists.

Practice – Remember **Think "Angle"**

Evaluate each in radians

$\cos^{-1}\left(\frac{1}{2}\right)$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\arccos(0)$

$\arctan\left(\frac{1}{\sqrt{3}}\right)$

$\arccos(0.9627)$

Practice – Use the triangle

Simplify:

$\sin\left(\cos^{-1}\left(1/\sqrt{5}\right)\right)$

$\tan(\cos^{-1}(x))$